QUINE ON LOGISTIC

A System of Logistic. By Willard Van Orman Quine. Harvard University Press, 1934. x+204 pp.

In this book is presented a system of symbolic logic based on that of Whitehead and Russell's *Principia Mathematica*, but involving a number of fundamental changes. The most important of these changes are: (1) the representation of functions of two or more variables as functions of one variable through the introduction, as an undefined term, of the operation of ordination, that is, the operation of combining two elements a and b into the ordered pair a,b; (2) the use of this same notion of ordination to replace the notion of predication, the proposition ϕa, obtained by predicating the propositional function ϕ of the argument a, being identified with the ordered pair ϕ,a; (3) the introduction in connection with the operation of abstraction, ^, of a rule of inference, the rule of concretion, which takes the place of that tacit rule of *Principia* which, to speak somewhat inexactly, allows the substitution for ϕx, in any proved expression in which ϕ is a free variable, of any appropriate expression containing x; (4) a liberalization of the theory of types, by which the axiom of reducibility is rendered unnecessary; (5) the use of the notion of classial referent, introduced by an actual nominal definition, to replace almost entirely the clumsy descriptions introduced in *Principia* as incomplete symbols; (6) the introduction, under the name of congeneration, of the relation of implication between propositional functions, as an undefined term, out of which both the relation of implication between propositions and the universal and existential quantifiers are obtained by definition.

Quine's propositional functions have the property that equivalence implies equality, and for this reason he speaks of them as classes rather than as propositional functions. Nevertheless he uses them for the purposes for which propositional functions are used in *Principia* and in other systems, and hence, for the sake of comparison, we continue to call them propositional functions.

In regard to Quine's use of ordination, it is, of course, clear, as he points out, that the introduction as primitive ideas of an infinite number of different notions of predication, one for functions of one variable, another for functions of two variables, another for functions of three variables, and so on, is awkward and that it is therefore desirable to find some device by which functions of two or more variables can be regarded as special cases of functions of one variable. It is not so clear, however, that the introduction of the ordered pair as an undefined term is the best method of doing this. From some points of view the more natural and more elegant method is that of Schönfinkel,* under which a function of $n+1$ variables is regarded as a function of one variable whose values are functions of n variables. For example, instead of what is ordinarily written $\phi(a, b)$, Schönfinkel writes $(\phi a)b$, where ϕa is regarded as a function which, when taken of the argument b, yields the proposition $(\phi a)b$, and ϕ is regarded as a function which, when taken of the argument a, yields the func-

* Mathematische Annalen, vol. 92 (1924), pp. 305–316.