A CONVERGENCE FACTOR THEOREM IN THE THEORY OF SUMMABLE SERIES*

BY H. L. GARABEDIAN

1. *Introduction.* The object of this paper is to write down sufficient conditions to ensure that any definition of summability of the *convergence factor type*† be more effective than or *include*‡ the definition of de la Vallée-Poussin.

A series

$$\sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + \cdots \tag{1}$$

is said to be summable by the method of de la Vallée-Poussin (or summable VP) to the sum S if $\lim_{n\to\infty} v_n = S$, where

$$\begin{aligned} v_n &= u_0 + \sum_{k=1}^{n} \frac{C_{2n,\, n-k}}{C_{2n,\, n}} u_k \\ &= u_0 + \sum_{k=1}^{n} \frac{n(n-1)\cdots(n-k+1)}{(n+1)(n+2)\cdots(n+k)} u_k. \end{aligned} \tag{2}$$

For the sake of convenience, we shall define the series (1) to be ϕ-summable to the sum S provided that the set of functions, $\phi_n(\alpha)$, $(n=0, 1, 2, \cdots)$, are defined for a set of values $E(\alpha)$ having at least one limit point α_0, not of the set; the series $\sum_{n=0}^{\infty}\phi_n(\alpha)u_n$ converges over $E(\alpha)$; and

$$\lim_{\alpha\to\alpha_0} \sum_{n=0}^{\infty} \phi_n(\alpha) u_n = S.$$

The theorem studied in this paper is stated as follows.

THEOREM. *If* (i) *the series* (1) *is summable VP to the sum S,* (ii) *the set of functions, $\phi_n(\alpha)$, $(n=0, 1, 2, \cdots)$, defined for a set of values $E(\alpha)$ having at least one limit point α_0, not of the set, satisfies the conditions*

---

* Presented to the Society, September 5, 1934.

† C. N. Moore, Transactions of this Society, vol. 8 (1907), p. 299.

‡ W. A. Hurwitz, this Bulletin, vol. 28 (1922), p. 17.