ON THE SUMMABILITY OF A CERTAIN CLASS OF SERIES OF JACOBI POLYNOMIALS*

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1. *Introduction.* The result obtained in this paper is as follows:

The series

$$\sum_{n=1}^{\infty} n^i \frac{(p+1)(p+3)\cdots(p+2n-1)}{2^n n!} X_n^{(p-1)/2}(x),$$

where $X_n^{(p-1)/2}(x)$ *(hereafter indicated simply by* X_n*) is a symmetric Jacobi polynomial,*† $p>-1$, *and* i *a positive integer, is summable* (C, k), $k>i-1/2$, *for the range* $-1<x<1$.

The proof is limited to symmetric Jacobi polynomials because of the necessity of having the recursion formula‡ of first degree in n. Unless explicitly stated otherwise, x is confined to the range $-1<x<1$, and $p>-1$, throughout this paper.

In the proof the sum of the n first terms of the given series is transformed, following the method employed by Brenke for Hölder summability of certain series of Legendre polynomials,§ by the recursion formula for Jacobi polynomials into a new sum of n terms, plus four additional terms. Then convergence factors for summability $(C, i-1)$ are applied, followed by those of summability (C, j), $j>1/2$, necessary to evaluate the additional

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† This is denoted by $F(-n, n+p, (p+1)/2, (1-x)/2)$ in the notation of Darboux, *Mémoire sur l'approximation des fonctions de très grands nombres*, Journal de Mathématiques, (3), vol. 4 (1878), pp. 5–60, 377–416; p. 22. It is $G_n(p, (p+1)/2, (1-x)/2)$ in the notation of R. Courant and D. Hilbert, *Methoden der Mathematischen Physik*, vol. 1, p. 74. It is $X_n^{((p-1)/2,\ (p-1)/2)}(x)$ in the notation of G. Pólya and G. Szegö, *Aufgaben und Lehrsätze aus der Analysis*, vol. 2, pp. 93–94, where the orthogonality property is expressed by means of the equality $\int_{-1}^{1}(1-x)^{(p-1)/2}(1+x)^{(p-1)/2}X_n^{((p-1)/2,(p-1)/2)}X_m^{((p-1)/2,(p-1)/2)}dx$ $=0(m\neq n;\ m, n=0, 1, \cdots)$.

‡ Darboux, loc. cit., p. 378.

§ W. C. Brenke, *On the summability and generalized sum of a series of Legendre polynomials*, this Bulletin, vol. 39 (1933), pp. 821–824.