SOME NEW COMPLETE SETS OF IDENTITIES FOR AFFINE AND METRIC SPACES

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1. *Introduction.* Complete sets of identities have been obtained for the components of the affine and metric normal tensors and also for the components of the affine, metric, and projective curvature tensors. In addition to these identities, a complete set is known for the components of the first covariant derivative of the affine curvature tensor.*

In this paper a complete set of identities is obtained for the components of the second covariant derivative of the affine curvature tensor, and also a complete set for the first and second covariant derivatives of the metric curvature tensor.

2. *The First Covariant Derivative.* We shall now obtain a complete set of identities for the components† $B_{ijkl,m}$ of the first covariant derivative of the metric curvature tensor with components B_{ijkl}. In terms of the g_{ij} and their derivatives, we have

$$B_{ijkl} = \frac{1}{2}\left(\frac{\partial^2 g_{ik}}{\partial x^j \partial x^l} - \frac{\partial^2 g_{il}}{\partial x^j \partial x^k} - \frac{\partial^2 g_{jk}}{\partial x^i \partial x^l} + \frac{\partial^2 g_{jl}}{\partial x^i \partial x^k}\right) + g_{ab}(\Gamma^a_{ik}\Gamma^b_{jl} - \Gamma^a_{il}\Gamma^b_{jk}). \tag{1}$$

By expressing (1) in metric normal coordinates, differentiating and evaluating at the origin, we obtain

$$B_{ijkl,m} = (g_{ik,jlm} + g_{jl,ikm} - g_{il,jkm} - g_{jk,ilm})/2. \tag{2}$$

The quantities $g_{ij,klm}$ are the components of the third extension of the g_{ij}. The following identities form a complete set for these components:

$$\begin{gathered} g_{ij,klm} = g_{ji,klm} = g_{ij,pqr}, \\ g_{ij,klm} + g_{ik,jlm} + g_{il,jkm} + g_{im,jkl} = 0, \end{gathered} \tag{3}$$

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* T. Y. Thomas, *Differential Invariants of Generalized Spaces*, Cambridge University Press, 1934, pp. 81, 114, 132, 138.

† In this section and the next small Latin indices have the range 1, 2, · · · , n.