SHORTER NOTICES

Einfuehrung in die Hoehere Analysis. By Ernst Lindelöf. German edition by Egon Ullrich. Leipzig, Teubner, 1934. ix+526 pp.

This volume fulfills in an able manner the promise of its title, that of giving an introduction to higher analysis, which may serve either as a text in a class, or be used for self-instruction. It reads like a series of lectures given by an able teacher with a wide acquaintance with the field presented and a long experience in the presentation of material. It might well serve as a model of exposition even for the teacher of calculus of long standing. Among the points to be noticed are the frequent occasions when the author presents a point from two or three different angles. It is further remarkable how well the author has succeeded in stating the fundamental theorems in the calculus in such a way as not to sacrifice in accuracy, and still be intelligible. The beginning of the chapter on the definite integral, where care is taken to define the meaning of length and area, is a case in point. On the other hand, where necessary, the author is content with stating the theorem and delaying proof until greater maturity has been reached, as for instance in properties of continuous functions, which are taken up in a chapter near the end of the book.

The beginning sections of the book bring, in addition to an admirable introduction to elementary functions, and the treatment of limits, a chapter on approximate computation, with special attention to the question of significant figures, and a chapter on continued fractions. The principal section of the book is devoted to the essentials of differential and integral calculus, with many delightful sidelights. Near the close of the book, the author indulges in a chapter on real numbers, which is really a fine introduction to the theory of functions of a real variable, and a chapter on complex numbers which includes the general solution of the cubic and biquadratic equations. An appendix brings an introduction to the theory of determinants, the presentation of which suggests the possibility of defining the expansion of determinants by expansions by minors, using an induction process for proof, a notion which might well have advantages for an introduction to the subject. Another appendix lays the foundation of the treatment of areas by developing the formula for the area of a closed polygon

$$\sum_{i=1}^{n} \tfrac{1}{2}(x_i y_{i+1} - x_{i+1} y_i); \quad n + 1 \equiv 1,$$

seldom found in analytic geometries, and a corresponding result for volumes of polyhedra.

One might take exception to the length of the volume, but the objective, the clearness and elegance of the treatment, make this item seem insignificant. After all, as the German editor remarks, it is not the paper and printer's ink which should be minimized, but the enjoyment of the ultimate consumer which should be maximized, and this ideal has certainly been attained in this volume.

T. H. HILDEBRANDT