TWO BOOKS ON TOPOLOGY

Vorlesungen über die Theorie der Polyeder unter Einschluss der Elemente der Topologie. By Ernst Steinitz; edited and completed by Hans Rademacher. Berlin, Springer, 1934. viii+351 pp.

Lehrbuch der Topologie. By H. Seifert and W. Threlfall. Leipzig and Berlin, Teubner, 1934. vii+353 pp.

These two books present an interesting contrast; both have to do with topology in a pure, or combinatorial form, but otherwise they have little in common. The one is a memoir; the other is a textbook. The one is an isolated chapter of geometry which bears little relation to the main stream of contemporary topological research, but which stands by itself, firm in its own intrinsic worth. It deals with a question almost as old as analysis situs itself—the combinatorial classification of ordinary polyhedra. The other is an exposition of the rudiments of modern topology—the homology theory of n-dimensional complexes and manifolds. It aims to give the student a glimpse of the far-reaching developments of present-day topology, and to acquaint him with the vital ideas at their root.

The book by Steinitz is a posthumous memoir, appearing as the forty-first volume of the familiar yellow-covered series put out by Springer. It carries on the study of ordinary (two-dimensional) polyhedra already begun by Steinitz in his *Enzyklopädie* articles on *Polyeder und Raumeinteilungen.* The theory develops about a single central problem: Are there necessary and sufficient combinatorial conditions in order that it be possible to realize geometrically a two-dimensional complex by a convex polyhedron? In other words, can convex polyhedra be characterized in a purely combinatorial fashion? This problem has a classical ring; it is a natural outgrowth of the pioneer work of Euler on convex polyhedra; yet here it receives for the first time a complete and definite affirmative answer. The book is self-contained, well-illustrated, and easy to read. One cannot go far in it without sensing the enthusiasm of an author who has made his subject a hobby as well as a serious piece of mathematics. Steinitz' fame may rest on his contributions to algebraic field-theory, but he evidently took just as much delight, if not more, in working with polyhedra.

The book is divided into three parts. The first part makes an exploration, partly historical and partly intuitive, of possible forms and types of polyhedra. It discusses the following topics: the Euler formula ($v-e+f=2$) and its extensions, polyhedral volume, the topological forms of surfaces (including the non-orientable), the Cauchy theorem on the rigidity of a convex polyhedron, and the Legendre determination of the number of degrees of freedom involved in the construction of a polyhedron of given type. At the end of this preliminary survey the problem of the combinatorial characterization of convex polyhedra is broached through polyhedra with triangular faces and the dual polyhedra with three-edged vertices.

The second part of the book presents a purely combinatorial theory of polyhedra, guided by the preceding exploration, but founded on strict abstract