A THEOREM CONCERNING LOCALLY PERIPHERALLY SEPARABLE SPACES*

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Alexandroff has shown that a connected metric space is completely separable provided it is locally completely separable.† It is the purpose of this paper to establish a similar theorem for connected, locally connected metric spaces.

DEFINITION. *A space is locally peripherally separable provided that, if P is a point of a region R, there exists in R a domain D containing P such that the boundary of D is separable.*

THEOREM. *A connected, locally connected, locally peripherally separable metric space is completely separable.*

PROOF. Suppose that n is a fixed positive integer. Let G denote the collection of all domains (to be called regions) of diameter $1/n$ or less which are peripherally separable. Since space is locally peripherally separable, it is evident that G covers space. It will now be shown that G contains a countable subcollection covering space. For each point X of space let n_x denote the largest integer such that no region of G contains a circular domain with center at X and radius greater than or equal to $1/n_x$. Let D_1 denote some region of G. For each integer i let M_{1i} denote the set of all points X of the boundary β_1 of D_1 such that $n_x = i$. Since space is metric and β_1 is separable , β_1 is completely separable, and there exists in M_{1i} a countable point set N_{1i} which is everywhere dense in M_{1i}. Now for each point X of N_{1i} let R_x denote a region of G containing a circular domain with center at X and radius $1/(i+1)$. The sum of these regions R_x forms a domain Q_{1i} covering M_{1i}, and $\sum_{i=1}^{\infty} Q_{1i}$ is a domain covering β_1.

Let $D_2 = D_1 + \sum_{i=1}^{\infty} Q_{1i}$. Then D_2 contains $D_1 + \beta_1$. Since space is locally connected, every point of the boundary β_2 of D_2 either belongs to the boundary of some region R_x or is a limit point of

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† Paul Alexandroff, *Über die Metrization der im kleinen kompakten topologischen Räume*, Mathematische Annalen, vol. 92 (1924), pp. 294–301.