A NEW PROOF OF MINKOWSKI'S THEOREM ON THE PRODUCT OF TWO LINEAR FORMS

BY R. Q. SEALE

Remak,* Mordell,† Landau,‡ and Blichfeldt§ have all proved the theorem, first proved by Minkowski‖ (1901):

If α, β, γ, δ, ξ_0, η_0 are real and $\alpha\delta-\beta\gamma=1$, integers x, y always exist such that

$$\left| (\alpha x + \beta y - \xi_0)(\gamma x + \delta y - \eta_0) \right| \leqq \frac{1}{4}.$$

This theorem includes as a special case, the classical theorem of Tchebychef¶ (1866):

If a is irrational and b is real, an infinite number of pairs of integers x, y $(y>0)$ always exist such that $|(x-ay-b)|$ can be made arbitrarily small, and at the same time

$$\left| x - ay - b \right| < \frac{2}{y}\cdot$$

In what follows, by making use of no principles more advanced than the elementary properties of convergents, I have proved three theorems, the first one being the Tchebychef theorem stated above. The second is Minkowski's theorem on the product of two homogeneous forms, while the third is the Minkowski theorem stated above. I feel that, although Tchebychef's theorem is a special case of Minkowski's theorem, its

---

* Bachmann, *Die Arithmetik der Quadratischen Formen*, Zweite Abteilung, p. 66; or Remak, Journal für die reine und angewandte Mathematik, vol. 142, p. 278.

† Mordell, Journal of the London Mathematical Society, vol. 3 (1928), p. 19.

‡ Landau, Journal für die reine und angewandte Mathematik, vol. 165 (1931), p. 1.

§ Published in a syllabus which Professor Blichfeldt distributed to a class in geometry of numbers at Stanford University, winter and spring quarters, 1932.

‖ Minkowski, *Diophantische Approximationen*, pp. 42–45.

¶ Œuvres de Tchebychef, vol. 1, p. 637.