A GROUP OF OPERATIONS ON A PARTIALLY COLORED MAP

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It has been previously shown* that if there be a minimum uncolorable map, and it is colored with the exception of one five-sided region in the four colors A, B, C, and D, then only one of the colors (say B) will be repeated in the five regions touching the region not colored, and that this ring of regions can be assigned the colors $DBABC$ in order. It has likewise been shown that when the map is thus colored, there will be two intersecting Kempe chains running from A to C and A to D, respectively. We shall say that any map, whether minimum uncolorable or not, which is partially colored in the above manner is colored *impasse*. The situation may be represented as in Fig. 1.

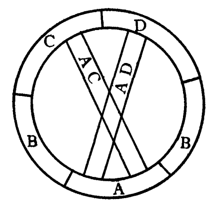

FIG. 1

The diagram does not attempt to show where or how many times the circuits cross one another. It merely means that they do cross. Here we have placed the uncolored five-sided region outside the rest of the map. We do this for compactness, but it must be noted that what follows is not dependent upon this choice of representation, nor upon the choice of lettering. Also to be remembered is the fact that right and left have only arbitrary meanings in discussing a map which may be deformed in any continuous manner over the surface of a sphere. Most writers would draw such a map with the uncolored region inside. We here call the D region the one in the diagram above on the right of A, and the C region the one on the left of A. We make the following definitions.

The region A is called the *vertex* of the ring. It is the region in this ring which lies between the two regions having the same color. We call the AC circuit the *left-hand circuit* and the AD circuit the *right-hand circuit*. We call the BD chain which includes the B region in the ring between A and C the *left-hand*

* P. J. Heawood, *Map colour theorem*, Quarterly Journal of Mathematics, vol. 24 (1890), pp. 332–338.