

Choose  $n$  to be a prime  $q$  represented by  $C$  and prime to  $d$ . Then  $g(q) = 2$  if  $C$  is ambiguous,  $g(q) = 1$  if  $C \neq C^{-1}$ . If a form  $f_1$  is associated with a form  $g_1$  not in  $C$  or  $C^{-1}$ ,  $f_1(p^2q) = \sigma g_1(q) = 0$ . Hence, by (2) and (3),  $p^2q$  is represented in exactly  $\eta \{p - (d' | p)\} \sigma^{-1}$  classes  $K$ , where  $\eta$  is 1 or 2 according as  $q$  is represented in only one (ambiguous) or two (reciprocal) primitive classes of discriminant  $d'$ .

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## ON A REDUCTION OF A MATRIX BY THE GROUP OF MATRICES COMMUTATIVE WITH A GIVEN MATRIX\*

BY P. L. TRUMP

1. *Introduction.* Two  $n \times n$  matrices  $A$  and  $B$ , with elements in any field  $F$ , are said to be similar in  $F$  if there exists a non-singular  $n \times n$  matrix  $S$ , with elements in  $F$ , such that  $S^{-1}AS = B$ .

Ingraham† has given a method for finding the most general solution, with elements in  $F$ , of the matrix equation

$$P(X) = A,$$

where  $P(X)$  is a polynomial with coefficients in  $F$ , and  $A$  is a square matrix with elements in  $F$ . A certain set of dissimilar solutions  $X_1, X_2, \dots, X_k$  were obtained in terms of which the complete system of solutions was seen to be in the form  $S^{-1}X_iS$ , where  $S$  is commutative with  $A$ . The  $X_i$ 's are obviously commutative with  $A$ .

The purpose of this investigation is to determine the conditions under which two  $n \times n$  matrices  $C$  and  $D$  are similar under transformations of the group  $[S]$  of non-singular matrices  $S$  which are commutative with a certain  $n \times n$  matrix  $A$ , where the matrices  $C$  and  $D$  are also commutative with  $A$ . We then seek to describe possible canonical forms to which such matrices

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\* Presented to the Society, September 4, 1934. This paper with proofs and detail that are omitted here, is on file as a doctor's thesis at the Library of the University of Wisconsin.

† *On the rational solutions of the matrix equation  $P(X) = A$* , Journal of Mathematics and Physics, vol. 13 (1934), pp. 46-50.