A NEW SOLUTION OF THE GAUSS PROBLEM ON $h(s^2d)/h(d)$*

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The following demonstration of the well known formula

$$h(p^2d') = \sigma^{-1}\{p - (d' \mid p)\}h(d') \tag{1}$$

may be worth noting. Here $h(\Delta)$ denotes the number of classes of primitive integral binary quadratic forms of non-zero discriminant Δ; p is any prime $\geqq 2$; $\sigma = 1$ if $d' < -4$ or d' is a square, $\sigma = 2$ if $d' = -4$, $\sigma = 3$ if $d' = -3$; and if d' is positive but not square, σ is the least positive integer for which $p \mid u_\sigma$, (t_k, u_k) denoting the successive positive integral solutions of $t^2 - d'u^2 = 4$.

Let $r(n)$ denote the number of sets of representations of n by a representative system of primitive forms of discriminant $d = p^2d'$. If q is a prime such that $(d \mid q) = 1$,

$$r(p^2q) = 2\{p - (d' \mid p)\}. \tag{2}$$

For by II (5), (33), (23)–(24),†

$$r(p^2q) = r(p^2)r(q) = 2r(p^2) = 2\{1 + r'(p^2)\},$$

where $r'(p^2)$ equals the number $p - 1 - (d' \mid p)$ of solutions w of

$$(pw)^2 \equiv p^2d' \pmod{4p^2}, \quad \frac{w^2 - d'}{4} \text{ prime to } p, \quad (0 \leqq pw < 2p^2).$$

By Theorem 4 of I, extended to $d > 0$ in II, there is associated with each class (connoted by K, say) of primitive forms f of discriminant p^2d', a unique ambiguous class C, or two non-ambiguous classes C and C^{-1}, of primitive forms g of discriminant d'; C is characterized as representing any prime represented by K. By II (13), such forms satisfy, for all integers n,

$$f(p^2n) = \sigma g(n). \tag{3}$$

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† References are to the writer's two papers: I, Mathematische Zeitschrift, vol. 36 (1933), pp. 321–343; and II, Transactions of this Society, vol. 35 (1933), pp. 491–509.