NOTE ON NON-ANALYTIC FUNCTIONS*

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In his retiring presidential address before the American Mathematical Society (December, 1931), E. R. Hedrick gave a resumé of the theory of non-analytic functions of a complex variable.† We propose in the present note to establish, with regard to these functions, some elementary properties that do not appear in the Hedrick report. (In the course of the work, however, some properties considered by Hedrick will present themselves.)

Let

$$(1)\qquad w = u + iv = f(z), \qquad z = x + iy,$$

where u, v have continuous first partial derivatives in x and y, in some plane region $\mathfrak{R}$. Equation (1) can be expressed as a point transformation in the plane:

$$(2)\qquad T\colon\quad u = u(x, y), \qquad v = v(x, y).$$

If f is non-analytic in $\mathfrak{R}$ (as we shall assume throughout), then T is not a *directly conformal* transformation; that is, the magnitude and sense of angles are not both preserved under T. Let m_1, m_2 be (the slopes of) any two directions at a point $z=x+iy$. In general they will transform into directions not forming the same angle (magnitude and sense both considered). In fact, the following theorem is easily verified by elementary methods.

THEOREM 1. *A necessary and sufficient condition that two distinct directions m_1, m_2 at a point $z=x+iy$ transform conformally‡ under T is that m_1 and m_2 satisfy the relation*

$$(3)\qquad (G - J)m_1m_2 + F(m_1 + m_2) + (E - J) = 0.$$

Here

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† *Non-analytic functions of a complex variable*, this Bulletin, vol. 39 (1933), pp. 75–96.

‡ By *conformal* we shall mean *directly conformal*.