TETRAHEDRA ASSOCIATED WITH CANONICAL EXPANSIONS FOR A CURVED SURFACE*

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A number of different canonical developments for the equation of a curved surface have been obtained by investigators in the field of projective differential geometry. These developments† are simplest in form when the vertices of associated tetrahedra are located on a certain quadric, known as the canonical quadric of Wilczynski.‡ The geometrical location of this quadric was accomplished by Wilczynski by a very complicated method. Bompiani§ has offered a distinctly different definition, and Stouffer||, using the general methods of Wilczynski, has found a rather simple method of locating the quadric.

The canonical quadric is actually useful in locating only one of the four vertices of the tetrahedron. It is the purpose of this paper to locate the fourth vertex for the whole series of expansions by rather elementary methods and without the introduction of Wilczynski's quadric. As a matter of fact, the quadric is located as soon as any one of these fourth vertices is determined.

We shall suppose that the asymptotic net is parametric, and take the fundamental differential equations in the form

$$(1) \qquad y_{uu} + 2by_v + fy = 0, \qquad y_{vv} + 2a'y_u + gy = 0.$$

Using the notation introduced in the celebrated memoir by Green we shall put

$$(2) \qquad \rho = y_u - \beta y, \qquad \sigma = y_v - \alpha y,$$

where α and β are functions of u and v which may be assigned as desired. If the point y on the surface and the corresponding points ρ and σ are chosen as three vertices of the tetrahedron

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† Green, Transactions of this Society, vol. 20 (1919), pp. 79–153.

‡ Wilczynski, Transactions of this Society, vol. 9 (1908), pp. 79–120.

§ Bompiani, Rendiconti Accademia Lincei, (6), vol. 6 (1927), pp. 187–190, and Mathematische Zeitschrift, vol. 29 (1929), pp. 678–683.

|| Stouffer, Proceedings of the National Academy of Sciences, (18), vol. 3 (1932), pp. 252–255.