ON THE INDEPENDENCE OF UNDEFINED IDEAS

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Considerable attention has been devoted recently to the independence of postulates, but little notice has been paid to the question of the independence of the undefined ideas occurring in postulate sets. It is true that Padoa* has given a definition of independence of undefined ideas and has constructed a test to show its presence; but, during the thirty-four years which have followed his work, no one, so far as I know, has discussed the concept or applied the test. I therefore feel that it may not be out of place again to define this concept and to attempt to justify the test for it. I also undertake to exhibit the importance of the independence of undefined ideas by showing the situation which arises when it is lacking.

Suppose S is an abstract mathematical system with undefined ideas $(K_1, K_2, \cdots; R_1, R_2, \cdots; \odot_1, \odot_2, \cdots)$, where K_i represents an undefined class, R_i an undefined relation, and $\odot_i$ an undefined operation. Then we say that any one of the classes, as K_1, is dependent on the other undefined ideas if there exists in S a theorem such as the following:

$$(1) \qquad (x \epsilon K_1) \equiv F(x; K_2, \cdots; R_1, R_2, \cdots; \odot_1, \odot_2, \cdots),$$

where the triple bar indicates mutual implication. Similarly we say that R_1 is dependent if there exists in S a theorem such as

$$(2) \qquad (xR_1y) \equiv G(x, y; K_1, K_2, \cdots; R_2, \cdots; \odot_1, \odot_2, \cdots),$$

and that $\odot_1$ is dependent if there exists in S a theorem such as

$$(3) \qquad (x\odot_1 y) = H(x, y; K_1, K_2, \cdots; R_1, R_2, \cdots; \odot_2, \cdots),$$

where the double bar indicates identity. (It is important to notice that, in (1), the only undefined symbol occurring on the left is K_1, and that K_1 does not occur on the right; similarly for R_1 and $\odot_1$, respectively, in (2) and (3)). If an undefined idea is not dependent, it is said to be independent.

* *Essai d'une théorie algébrique des nombres entiers, précédé d'une introduction logique à une théorie déductive quelconque*, Bibliothèque du Congrès International de Philosophie, vol. 3 (1900).