PRIME ENDS AND INDECOMPOSABILITY*

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1. *Introduction.* Under some circumstances the set of the prime ends‡ of a plane bounded simply connected domain includes one which contains all the boundary points of the domain. This paper will establish a sufficient condition for the existence of such a prime end, and also a necessary condition. One of the arguments to be given shortly will require a special sequence of simple closed curves. The following informal development will establish satisfactorily the existence and chief properties of the sequence.

Let γ be a simply connected bounded plane domain containing the continuum G and having boundary Γ. Let ϵ be any preassigned positive number. There exists a class G_1^ϵ of simple closed curves such that if C is any one of G_1^ϵ and c is any point of C, then§ $\gamma \supset i(C) \supset G$, and $d(c, \Gamma) < \epsilon$. There exists a subclass G_2^ϵ of G_1^ϵ such that if C is any one of G_2^ϵ and g_u is any point of $\Gamma + \gamma \cdot e(C)$ and g_v any point of γ such that $d(g_v, \Gamma) > \epsilon$, then $d(g_u, C) < \epsilon$ and $i(C) \supset g_v$. If $[p_h]$, $(h = 1, 2, \cdots, m)$, is any finite subset of the points of Γ, there is a subclass G_3^ϵ of G_2^ϵ such that, if C is any one of G_3^ϵ, then $e(C) \supset \sum p_h$. There is a subclass G_4^ϵ of G_3^ϵ such that if C is any one of G_4^ϵ, then $C \cdot \Gamma$ is a finite nonvacuous set of points $[c_g]$, $(g = 1, 2, \cdots, n)$, and if K is any component of $C - \sum c_g$, then $d(K) < \epsilon$. If $[P_{hk}]$, $(h = 1, 2, \cdots, m$ and $k = 1, 2)$, is a set of $2m$ arcs with end points $[p_{hk}, c_{hk}]$, respectively, such that $\gamma \supset \sum (P_{hk} - p_{hk})$, and if $p_{hk} = p_h$, $(k = 1, 2)$, for all values of h, and if $\gamma \cdot P_{hk} \cdot P_{h'k'} = 0$ for all admissible values of h, k, h', and k' except of course when $h = h'$ and $k = k'$;

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‡ Defined by C. Carathéodory in his paper, *Über die Begrenzung einfach zusammenhängender Gebiete*, Mathematische Annalen, vol. 73 (1912), pp. 323–370.

§ If K is a point set then $c(K)$ represents its complement in space. If C is a simple closed curve in the plane then $i(C)$ represents its interior and $e(C)$ its exterior. Combining the symbols gives $ce(C) = C + i(C)$ and $ci(C) = C + e(C)$.