IRREDUCIBILITY OF POLYNOMIALS OF DEGREE n WHICH ASSUME THE SAME VALUE n TIMES*

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1. *Introduction*. A polynomial $F(x)$ of degree n, with integral coefficients, which assumes the same value k for n distinct integral values of x has the form

$$F(x) = a_0(x - a_1)(x - a_2) \cdots (x - a_n) + k, \qquad (a_0 \neq 0),$$

where the a's denote integers, and $a_1, a_2, \cdots, a_n$ are distinct. The irreducibility of polynomials of this type in the field of rational numbers has been discussed by several writers for the particular cases† $|k| = 1$, $|k| = \text{prime}$.

The present paper is concerned with the irreducibility of $F(x)$ for the case in which k is any integer $\neq 0$. It is obvious that even when the a's are fixed, an infinitude of choices of k exists for which $F(x)$ is reducible. What is not obvious is that when k and n are fixed, *only a finite number of non-equivalent reducible polynomials of the form* $F(x)$ *exist*. Two polynomials $F(x)$ and $G(x)$, with integral coefficients, are regarded as *equivalent* if an integer h exists such that $F(x) = \pm G(\pm x + h)$. Moreover, if only k is fixed, but n is sufficiently large, *every* polynomial of the form of $F(x)$ is irreducible.

2. *Isolation of the Roots of* $f(x)$. The polynomial $F(x)$ of §1 is evidently equivalent to the polynomial

$$f(x) = ax(x - t_1) \cdots (x - t_{n-1}) \pm k,$$

where $a, k, t_1, \cdots, t_{n-1}$ are positive integers, and the t's are distinct. We shall confine our attention to $f(x)$ and assume that $n \geqq 2$. We shall denote by x_0 a root of $f(x)$ whose absolute value is a minimum, and the other roots by $x_1, \cdots, x_{n-1}$. Taking the ratio of the coefficient of x to the constant term in each of the last two members of

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† For literature, see Dorwart and Ore, Annals of Mathematics, vol. 34 (1933), p. 81; A. Brauer, Jahresbericht der Deutscher Mathematiker Vereinigung, vol. 43 (1933), p. 124.