$$\sum_{n=1}^{\infty} a_{mn}x_n, \qquad (m = m_1, m_2, \cdots),$$

converges. This however cannot occur if $\mathfrak{A}$ is regular in the sense of the definition above, according to which all series

$$\sum_{n=1}^{\infty} a_{mn}x_n, \qquad (m \geqq m'(x)),$$

must converge.

The existence of a fixed m_0 is thus established, and at the same time it is shown that our modified definition of regularity of $\mathfrak{A}$ is equivalent to the classical one. It is clear that analogous considerations can be applied when instead of summation of series we deal with summation of integrals.

BROWN UNIVERSITY

AN EXTENSION TO POLYGAMMA FUNCTIONS OF A THEOREM OF GAUSS*

BY H. T. DAVIS

1. *Introduction.* By the polygamma functions we mean the derivatives of log $\Gamma(x)$, that is, $\Psi(x) = \Gamma'(x)/\Gamma(x)$, $\Psi'(x), \cdots, \Psi^{(n)}(x)$.† These functions satisfy the difference equations:

$$\begin{gathered}\Psi^{(n)}(x+1) - \Psi^{(n)}(x) = (-1)^n \frac{n!}{x^{n+1}}, \\ \Psi^{(n)}(1-x) + (-1)^{n+1}\Psi^{(n)}(x) = A_n(x), \quad A_n(x) = \frac{d^n}{dx^n}(\pi \operatorname{ctn} \pi x),\end{gathered} \tag{1}$$

subject to the boundary condition,

$$\Psi^{(n)}(1) = (-1)^{n+1} n! S_{n+1},$$

where we employ the abbreviation

$$S_m = 1 + 1/2^m + 1/3^m + \cdots .$$

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† The name polygamma is suggested by the paper, *Tables of the Digamma and Trigamma Functions*, by Eleanor Pairman, Tracts for Computers, No. 1, 1919.