# ON THE NOTION OF REGULARITY OF METHODS OF SUMMATION OF INFINITE SERIES

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Let $\mathfrak{A}=(a_{mn})$ be the matrix of a method of summation which consists in replacing a given sequence $x=(x_1, x_2, \cdots)$ by its *transform*

$$(1) \qquad y = (y_1, y_2, \cdots), \quad y_m = \sum_{n=1}^{\infty} a_{mn}x_n, \qquad (m = 1, 2, \cdots),$$

and defining the generalized limit of $x_n$ as $\lim_{m\to\infty} y_m$, provided this limit exists.

The method $\mathfrak{A}$ is called regular if every convergent sequence $x=\{x_n\}$ is transformed into a convergent sequence $y=\{y_m\}$ with the same limit. Necessary and sufficient conditions for regularity of $\mathfrak{A}$ are too well known to be restated here. An essential point in the whole theory is the assumption that the sequence $y\equiv y(x)$ is determined by formula (1) for *each* convergent sequence $x$. A (quite trivial) gain in generality may be achieved by demanding that not all the terms $y_m$ of the sequence $\{y_m\}$ have to be considered but only those for which $m\geqq m_0$, where $m_0$ is a fixed integer. Even this requirement is not at all necessary, however, for the possibility of evaluating $\lim_{m\to\infty} y_m$, for which we have to know only *almost all* terms of the sequence $\{y_m\}$, that is all $y_m$, $m\geqq m'$, where $m'$ need not be fixed, but on the contrary, may depend on the sequence $x$.

Thus we are naturally led to the following apparently less restrictive definition of regularity of the method of summation $\mathfrak{A}$.

*The method of summation* $\mathfrak{A}$ *is regular if* (i) *to every convergent sequence* $x=\{x_n\}$ *there corresponds an integer* $m'(x)$ *such that* $y_m$ *as given by* (1) *exists for* $m\geqq m'(x)$, *and* (ii) *for a fixed* $x$,

$$\lim_{m\to\infty} y_m = \lim_{n\to\infty} x_n.$$

It turns out, however, that the modified definition of regularity

* The result of this note answers a question raised by Dr. H. Lewy.