ELEMENTARY DERIVATION OF THE EQUIVALENCE OF MASS AND ENERGY*

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The special theory of relativity grew out of the Maxwell electromagnetic equations. So it came about that even in the derivation of the mechanical concepts and their relations the consideration of those of the electromagnetic field has played an essential role. The question as to the independence of those relations is a natural one because the Lorentz transformation, the real basis of the special relativity theory, in itself has nothing to do with the Maxwell theory and because we do not know the extent to which the energy concepts of the Maxwell theory can be maintained in the face of the data of molecular physics. In the following considerations, except for the Lorentz transformation, we will depend only on the assumption of the conservation principles for impulse and energy.

We begin by making plausible the expressions for impulse and energy of the material particle in the well known way. The fundamental invariant of the Lorentz transformation is

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2,$$

or

$$ds = dt(1 - u^2)^{1/2},$$

where

$$u^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = u_1^2 + u_2^2 + u_3^2.$$

If one divides the components of the contravariant vector (dt, dx, dy, dz) by ds, one obtains the vector

$$\left(\frac{1}{(1 - u^2)^{1/2}}, \frac{u_1}{(1 - u^2)^{1/2}}, \frac{u_2}{(1 - u^2)^{1/2}}, \frac{u_3}{(1 - u^2)^{1/2}}\right).$$

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