statement is obviously true; if U_1 lies in $[U_1]$, $A = U_1 U_1^* P_1 U_1$ and, since $U_1^* P_1 U_1$ is a positive hermitian matrix of rank r, $U_1^* P_1 U_1 = P_2$ and U_1 lies in $[U_2]$. Similarly any member U_2 of $[U_2]$ lies in $[U_1]$. Further *the matrix P_2 is invariant under unitary transformation by any matrix of the group G_1, and P_1 under transformation by any matrix of the group G_2.* For if Z_1 lies in G_1, $AZ_1 = A$ so that $A = U_2 Z_1 Z_1^* P_2 Z_1$, and accordingly, $Z_1^* P_2 Z_1 = P_2$.

THE JOHNS HOPKINS UNIVERSITY

ON A THEOREM OF FÉRAUD

BY D. C. LEWIS, JR.*

The Birkhoff-Pfaffian equations of dynamics are written in variational form as follows:

$$\delta \int \left[\sum_{i=1}^{2m} X_i \left(\frac{dx_i}{dt} \right) + Q \right] dt = 0,$$

where Q and the X's are functions of $x_1, \cdots, x_{2m}$ and, in general, depend also periodically upon t, and where the skew-symmetric determinant $|a_{ij}|$, $(a_{ij} = \partial X_i/\partial x_j - \partial X_j/\partial x_i)$, does not vanish in the regions considered. We restrict attention to the neighborhood of a generalized equilibrium point, that is, a point where all the $\partial Q/\partial x_i - \partial X_i/\partial t$ vanish identically in t. We take this point at the origin, $x_i = 0$, $(i = 1, 2, \cdots, 2m)$.

The problem of reducing the Pfaffian system to a Hamiltonian system can be reduced to that of finding a non-singular transformation, $x_i = x_i(y_1, \cdots, y_{2m})$, leaving the origin invariant (and depending in general periodically upon t) which reduces the linear differential form $\sum_{i=1}^{2m} X_i dx_i$ to the form $\sum_{i=1}^{m} y_{2i} dy_{2i-1} + dw$, where dw is an exact differential in $y_1, \cdots, y_{2m}$, the coefficients of which are independent of t. This same problem also will play an important role in a future paper of mine on "conservative" transformations in $2m$-dimensional spaces.

The problem has been considered by Féraud,† who obtained a

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* National Research Fellow.

† *Extension au cas d'un nombre quelconque de degrês de libertê d'une propriêtê relative aux systèmes Pfaffiens*, Comptes Rendus, vol. 190 (1930), pp. 358–360.