DEGREE OF APPROXIMATION BY POLYNOMIALS TO CONTINUOUS FUNCTIONS*

BY W. E. SEWELL

1. *Introduction.*† This paper is primarily concerned with functions analytic in a given region and continuous in the corresponding closed region; the region is in all cases bounded by an analytic Jordan curve with no double points. The existence of polynomials which converge uniformly to the function in the closed region has been established by Walsh‡ for more general regions, but it is frequently convenient to have more precise results in the study of functions on the boundary of the region. The method used here is essentially a reduction of the problem to the expansion of a function in a Fourier series and consequently the function is assumed to satisfy a Lipschitz or Hölder condition. An extension of the classical theory of Fourier series, identification of this expansion with the Taylor expansion on the circumference of the unit circle, and a study of the degree of approximation of Faber's polynomials belonging to the region form the basis of this investigation.

2. *Relation between the Fourier and Taylor Expansions.* The following theorem can be proved easily by separating the function into its real and imaginary parts and examining the coefficients.

THEOREM 1. *Let $F(x)$ be analytic in $|x|<1$, continuous in $|x|\leqq 1$. Then the Taylor development for $F(x)$ about $x=0$ is precisely the Fourier development for $F(x)$ on the circumference, $|x|=1$.*

If $F(x)$, besides being continuous in the closed circle, satisfies

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‡ J. L. Walsh, Mathematische Annalen, vol. 96 (1926), pp. 430–436 and pp. 437–450.