

THE WEB OF QUADRIC HYPERSURFACES IN r DIMENSIONS*

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1. *Introduction.* Webs of quadric surfaces were defined and partially investigated in 1862 by de Jonquières. † Later, quadric webs were treated by many mathematicians, among whom were Cremona, Reye, Steiner, and Sturm. ‡ Still later, the involution defined by a web of quadrics was studied by Snyder and Sharpe. § There has been no treatment of webs in a space of higher dimension than three. The purpose of this paper is to derive the properties of a web of algebraic, quadric hypersurfaces in r dimensions.

2. *The Web.* The equation of a web of quadric hypersurfaces in S_r is

$$\sum \lambda_i f_i = 0, \quad (i = 1, 2, 3, 4),$$

in which the f_i are entirely general quadratic functions of x_1, x_2, \dots, x_{r+1} . The web has a basis manifold M_{r-4}^{16} of order 16 and dimension $r-4$. A doubly infinite non-linear system of quadric hypersurfaces of the web exists such that each hyperquadric of the system has a hypernode, that is, is a quadric hypercone. Also, among the hyperquadrics of the web, there is a doubly infinite non-linear system of pencils of hyperquadrics such that all the hyperquadrics of any one pencil have simple contact at one point. The locus of all such contacts as well as the locus of the hypernodes or vertices of the quadric hypercones of the web, is a surface J , the jacobian of the web. The characteristics of J will be found in §3.

The web also contains two singly infinite non-linear systems of pencils of hyperquadrics such that all the hyperquadrics in

* Presented to the Society, March 30, 1934.

† Journal de Mathématiques, (2), vol. 7 (1862), p. 412.

‡ Pascal, Repertorium der höheren Mathematik, vol. II₂ (1922), pp. 629–631. Encyklopädie der Mathematischen Wissenschaften, vol. III₂, second half, pp. 250–254.

§ Virgil Snyder and F. R. Sharpe, *Space involutions defined by a web of quadrics*, Transactions of this Society, vol. 19 (1918), pp. 275–290.