through the center of gravity and the vertices and so is a sphere.

The statement of our main theorem can be given in more general form but our statement is chosen on account of its intuitive simplicity. The set R we may take as merely closed and bounded; S may be the frontier of a bounded domain, D, which contains R. Then the conclusion remains the same as we have stated it in the simpler case.

THE RICE INSTITUTE

A DECOMPOSITION THEOREM FOR CLOSED SETS*

BY G. T. WHYBURN

Let P be any local† topological property of a closed set such that if K is any compact closed set lying in a metric space, then the set of all non-P-points of K is either vacuous or such that its closure is of dimension >0. The following are examples of such properties: (i) local connectivity, (ii) regularity (Menger-Urysohn sense), (iii) rationality, (iv) being of dimension $<n$, (v) belonging to no continuum of convergence, (vi) belonging to no continuum of condensation. In fact, it will be noted that in each of these cases, every non-P-point of a compact set K lies in a non-degenerate continuum of non-P-points of K. We proceed to prove the following theorem.

THEOREM. *If N denotes the set of all non-P-points of a compact closed set K in a metric space and if K is decomposed upper semi-continuously‡ into the components of $\overline{N}$ and the points of $K-\overline{N}$, then every point of the hyperspace H is a P-point of H.*

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† For the purposes of the present paper we shall understand by a local property of a set K a point property P such that if some neighborhood of a point x in K has property P at x, then K has property P at x; and conversely, if K has property P at x, then any neighborhood of x in K also has property P at x. A point x of K will be called a P-point or a non-P-point of K according as K does or does not have property P at x.

‡ For the notions relating to upper semi-continuous decompositions and for a proof that our particular decomposition is upper semi-continuous, the reader is referred to R. L. Moore, *Foundations of Point Set Theory*, American Mathematical Society, Colloquium Publications, 1932, Chapter 5.