# IMPLICIT FUNCTIONS OF ALMOST PERIODIC FUNCTIONS*

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1. *Introduction.* It is a well known fact that a uniformly continuous single-valued function of a uniformly almost periodic (u.a.p.) function is u.a.p., and that its module is contained in the module of the original function. However, the same statements cannot be made concerning multiple-valued functions; for instance, a u.a.p. function can be constructed which is never zero and whose two continuous square root functions are neither of them u.a.p. (or even Stepanoff a.p.). Moreover, when a square root (or other multiple-valued) function is a.p., it need not have a module contained in the module of the original. This is illustrated by $e^{ix}$ whose module consists of all integers, but whose square roots $e^{ix/2}$ and $-e^{ix/2}$ have modules consisting of all integers and half integers.

It is the aim of this paper to obtain sufficient conditions under which multiple-valued functions (implicit functions) of an a.p. function are a.p., and to investigate the module of such solutions. In the main, u.a.p. functions will be considered, but some results will also be obtained for S.a.p. functions.

2. *Statement of Theorem* 1. In the following theorem, $x$ denotes the real variable while $f(x)$, $z$, and $F(x, z)$ are either all real or all complex.

THEOREM 1. *Let $f(x)$ be a continuous bounded function defined for all values of $x$; let $\mathfrak{C}$ be the set of values which $f(x)$ can assume and let $\mathfrak{S}$ be the set of ordered pairs $(x, z)$ such that $|f(x)-z|<\mu$, $\mu>0$. Let $F(x, z)$ be a function having the following properties*:

(a) *$F(x, z)$ is u.a.p. in $x$ uniformly in $z$ for all $z$ in $\mathfrak{C}$.*

(b) *$F(x, z)$ is uniformly continuous in $z$ uniformly with respect to $x$ for all $(x, z)$ on $\mathfrak{S}$.*

(c) *$\partial F/\partial z = F_z(x, z)$ exists everywhere on $\mathfrak{S}$.*

(d) *$F[x, f(x)]=0$ and $|F_z[x, f(x)]|>\lambda>0$ for all $x$.*

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