CLASSES OF MAXIMUM NUMBERS ASSOCIATED WITH CERTAIN SYMMETRIC EQUATIONS IN n RECIPROCALS*

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1. *Introduction.* In an article dealing with this subject Simmons† stated without proof two general theorems whose proofs are to be obtained by making certain modifications in the theory of his article, which will be referred to in the sequel as I (for paper I). These theorems will be stated after a few definitions from I are recalled.

Kellogg solution. If a solution $x\equiv(x_1, x_2, \cdots, x_n)$ of any given symmetric equation in n reciprocals is obtained by minimizing the variables $x_1, x_2, \cdots, x_{n-1}$ (all positive integers) in this order, one at a time, we shall denote it by w and call it the Kellogg solution of the given equation. Thus $x\equiv(2, 3, 6)$ is the Kellogg solution w of the equation $x_1^{-1}+x_2^{-1}+x_3^{-1}=1$.

E-solution. A solution $x\equiv(x_1, x_2, \cdots, x_n)$ of a given symmetric equation in n reciprocals is called an E-solution if $x_1, x_2, \cdots, x_{n-1}$ are positive integers and $x_1\leqq x_2\leqq \cdots \leqq x_n$.

Polynomial $P(x)$. Let $P(x_1, x_2, \cdots, x_n)\equiv P(x)$ be any polynomial which is symmetric in the n variables x_i, contains one or more positive coefficients and no negative coefficient, and is not identically a constant.

$\Sigma_{i,j}(x)$. With $i\geqq 0$ and j equal to integers, we let $\Sigma_{i,j}(x)$ stand for the jth elementary symmetric function of the i variables $x_1, x_2, \cdots, x_i$; with the customary understanding that

$$\Sigma_{i,j}(x)\begin{cases}\equiv 0 \text{ when } i < j \text{ and also when } j < 0,\\ \equiv 1 \text{ when } j = 0.\end{cases}$$

We now state the two theorems referred to above with the numbering of I.

THEOREM 4. *If in the equation*

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† See H. A. Simmons, Transactions of this Society, vol. 34 (1932), pp. 876–907.