THE PRINCIPAL MATRICES OF A RIEMANN MATRIX*

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1. *Introduction.* A matrix ω with p rows and $2p$ columns of complex elements is called a *Riemann matrix* if there exists a rational $2p$-rowed skew-symmetric matrix C such that

$$(1) \qquad \omega C\omega' = 0, \qquad \pi = i\omega C\bar{\omega}'$$

is positive definite. The matrix C is called *a principal matrix* of ω and it is important in algebraic geometry to know *what are all principal matrices of ω in terms of a given one.* In the present note I shall solve this problem.

2. *Principal Matrices.* A rational $2p$-rowed square matrix A is called a projectivity of ω if

$$(2) \qquad \alpha\omega = \omega A$$

for a p-rowed complex matrix α. The Riemann matrices ω have recently† been completely classified in terms of their projectivities; so we may regard all the projectivities A of ω as known.

A projectivity A is called symmetric if $CA'C^{-1}=A$. Let A be a symmetric projectivity so that if $B=AC$, then $B'=(AC)' = -CA' = -AC = -B$ is a skew-symmetric matrix. Then iAC is Hermitian and so must be

$$(3) \qquad \delta = \omega(iAC)\bar{\omega}' = \alpha(i\omega C\bar{\omega}') = \alpha\pi.$$

Now π is positive definite so that $\pi=\rho\bar{\rho}'$, where ρ is non-singular. Then $\pi^{-1}=(\bar{\rho}')^{-1}\rho^{-1}=\bar{\sigma}'\sigma$ with σ non-singular. Hence $\alpha=\delta\pi^{-1}=\delta\bar{\sigma}'\sigma$ and

$$(4) \qquad \sigma\alpha\sigma^{-1} = \sigma\delta\bar{\sigma}'.$$

The matrix $\sigma\delta\bar{\sigma}'$ is evidently Hermitian and it is well known that then $\sigma\delta\bar{\sigma}'$ and the similar matrix α have *only simple ele-*

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† See my paper *A solution of the principal problem in the theory of Riemann matrices,* Annals of Mathematics, October, 1934.