ON THE ENUMERATION OF MAGIC CUBES*

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1. *Introduction.* Assume the cube with one corner at the origin and the three edges at that corner as axes of reference. In a magic cube the sum of the numbers in any line parallel to any one of the three axes is the same, and since the sum of all the numbers up to N^3 is $N^3(N^3+1)/2$, the sum in each of the rows must be $N(N^3+1)/2$. The square array in any plane parallel to a face of the cube forms a magic square according to the more general definition of a magic square as one in which the sum in each row and column is the same even if the numbers involved are other than the numbers from 1 to N^2 inclusive.

2. *Transformations.* We will call the square array in any plane parallel to a face of the cube a *slab*. It is clear that any permutation of a set of parallel slabs among themselves will not affect the magic property of a magic cube. There are manifestly $(N!)^3$ of these permutations, and by means of them we can bring any element into any desired cell of the cube. In particular we can bring the element N^3 to the origin, and after that we can, by a further permutation of the slabs, arrange the elements which lie on the three axes so that they read in descending order of magnitude from the origin out.

Again, by a rotation of the cube through an angle of 120° about the diagonal through the origin, we may make a cyclic permutation of the three axes. We may then assume that the element next to the origin on the x axis is larger than either of those next to the origin on the other two axes. Further, by a reflection of the whole cube in the xy plane followed by a rotation about the x axis of 90°, we may interchange the elements of the y and z axes without disturbing those on the x axis. We may then assume that the element next to the origin on the y axis is larger than that next to the origin on the z axis. We then define a *normal* cube as one which has the element N^3 at the origin with the elements on the three axes arranged in descending order of magnitude from the origin out, and also such

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