IV. *Any phrase-equation whose members have like initial letters is interderivable with a phrase-equation whose members are free of parentheses and have like initial letters.*

From the definition of the "terms" of a phrase, it is clear that if a phrase is free of parentheses all its terms are letters. Hence, by II, any phrase-equation E whose members are free of parentheses is interderivable with any phrase-equation resulting from permutation of non-initial letters within members of E. Therefore any phrase-equation whose members are free of parentheses and have like initial letters is interderivable with an equation of canonical form. It then follows, by III and IV, that every phrase-equation is reducible to canonical form.

In view of §§3–4, this concludes the proof that upon elimination of abbreviations all homogeneous linear identities with rational coefficients are generable by (R) and (R′) from (A) and (B).

HARVARD UNIVERSITY

A CHARACTERISTIC PROPERTY OF SURFACES OF NEGATIVE CURVATURE†

BY E. F. BECKENBACH

1. *Introduction.* Let there be given a piece of surface S in a representation

$$(1)\quad S:\quad x = x(u, v),\ y = y(u, v),\ z = z(u, v),\ u^2 + v^2 < \rho^2,$$

with the following properties.

(a) $x(u, v), y(u, v), z(u, v)$ have continuous partial derivatives of the third order.

(b) The representation is isothermic; that is to say, $E=G$, $F=0$, where

$$E = x_u^2 + y_u^2 + z_u^2, F = x_u x_v + y_u y_v + z_u z_v, G = x_v^2 + y_v^2 + z_v^2,$$

the subscripts denoting differentiation.

We put $E=G=\lambda(u, v)$. Then $\lambda(u, v) \geqq 0$, and the representation is conformal except at points where $\lambda(u, v)=0$. The cus-

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