are not both even. If a is even and b odd, then b, $b+2$, $a+b$ are the only odd numbers in (8). Hence a and b are both odd. Then the only odd numbers (8) are

$$a, b, a + 2, b + 2. \tag{9}$$

Hence $m \neq 10$. Thus $m=8$. Then (9) are congruent to 1, 3, 5, 7 in some order. By their sums, $a+b \equiv 2 \pmod 4$, whence $a \equiv b$. Hence $b \equiv a+4 \pmod 8$.

THEOREM 5. *If m is even and >8, there is no 3-set* (7). *If $m=8$,* (7) *is a 3-set if and only if a, b is one of the pairs* 1, 5 *or* 3, 7. *The same result holds if we replace* $2+mx$ *by* $6+mx$ *in* (7).

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HYPERTRANSCENDENTAL EXTENSIONS OF PARTIAL DIFFERENTIAL FIELDS*

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1. *Introduction.* In the abstract or formal theory of ordinary algebraic differential equations,† the concept of *differential field* as defined by Baer‡ has a role analogous to that of field in abstract algebra.§ A differential field is a commutative field closed with respect to a formally defined operation called differentiation. The defining rules for differentiation are taken from the elementary properties of derivatives of functions of a single variable. In this paper, with an abstract theory of partial differential equations in mind, we define *partial differential fields*, selecting the defining rules for differentiation from the elementary properties of partial derivatives of functions of several variables.

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† Raudenbush, *Differential fields and ideals of differential forms*, Annals of Mathematics, vol. 34 (1933), pp. 509–517; and *Ideal theory and algebraic differential equations*, Transactions of this Society, vol. 36 (1934), pp. 361–368. Also, O. Ore, *Formale Theorie der linearen Differentialgleichungen*, Journal für Mathematik, vol. 167 (1933), pp. 221–234, and vol. 168 (1934), pp. 233 252.

‡ R. Baer, *Algebraische Theorie der differentierbaren Funktionenkörper* I, Heidelberger Sitzungsberichte, 1927–28.

§ For the terms and theorems of abstract algebra, see van der Waerden, *Moderne Algebra*.