# MAGIC CIRCLES

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If at every one of the $n^2+1$ points defined by $n$ concentric circles and $n$ radii is placed one of the first $n^2+1$ integers such that every circle with its center carries the same sum as every radius, I shall call this figure a system of magic circles of order $n$. Every magic square of order $n$ gives rise to two such circular systems, for the square with its $n$ rows and $n$ columns may be transformed into concentric circles, the rows becoming the circles and the columns the radii; all that is necessary is to assign to the center $C$ the number $n^2+1$ or 1; in the latter case all integers of the magic square must be raised by unity. Such systems I shall exclude here and all other systems I shall then call true circular systems.

The circles will be denoted by letters $a, b, c, \cdots$, ($a$ being the smallest circle), and the radii by the integers $1, 2, 3, \cdots$ so that, for instance, the integer standing at the intersection of circle $d$ with radius 6 will be denoted by $d_6$.

All systems obtained by an interchange of rows, columns, or row and columns in any particular system will form a group and that system in the groups with $a_1=1$, $a_2<a_3<a_4< \cdots$, $b_1<c_1<d_1< \cdots$, and $b_1<a_2$ will be considered the representative of this group. Every group contains, then, $2(n!)^2(1/n)$ distinct systems and just one representative, because every system can belong to only one group and two systems are considered to be identical, if one may be obtained from the other by rotating merely the paper on which the figure is drawn. The sum of all integers in a system of order $n$ is

$$S_n = \sum_{i=1}^{n^2+1} i = \tfrac{1}{2}(n^2+1)(n^2+2),$$

and the magic sum is

$$s_n = (1/n)\left[S_n + (n-1)M\right] = 1 + b_1 + c_1 + d_1 + \cdots,$$

where $M$ is the element at the center, so that we have now two restrictions on $M$, namely, $1 < M \leqq n^2$, and $M \equiv 1 \pmod{n}$. For each $M$, all possibilities for the set $(b_1, c_1, d_1, \cdots)$ must be