HOPFNER ON GEODESY

Physikalische Geodäsie. By F. Hopfner. Mathematik in Monographien und Lehrbüchern. No. 15. Leipzig, Akademische Verlagsgesellschaft, 1933. ix+434 pp.

The type of textbook on geodesy most familiar to students in this country is a sort of practical manual of large-scale higher surveying. Often considerable attention is given to the details of observation. The point of view of such books is predominantly geometric; naturally there is little space for extended theoretical developments, often the proofs of formulas have to be omitted.

More theoretical is the type of book best exemplified by the classical treatises of Helmert and Clarke. In these there is more of theory and a nearer approach to mathematical rigor, also less attention to observational detail, but the points of view of the engineer in the field and of the computer or the theorist are not entirely forgotten. This comprehensiveness of outlook is one reason why these two works retain their value fifty years and more after their publication.

Hopfner's book takes us further over into the theoretical side than either Clarke or Helmert, or indeed than any book known to the reviewer calling itself a treatise on geodesy. This is said by way of information, not of blame; the author is entitled to choose his own treatment and the first eight chapters are well done within the limitations of the space available, though the question might arise as to how much of the material of these chapters is really geodesy, or to what extent it prepares the way for a study of geodesy.

Chapter I treats of the relation of line and surface integrals, Green's theorem, the Laplacian operator, and elliptic coordinates. Chapter II deals with spherical harmonics; Chapter III with Lamé's functions; and Chapter IV with attraction and potential, and also with Green's functions and Dirichlet's principle. Chapter V, a brief one, deals with the attraction and potential of a homogeneous ellipsoid. In Chapters VI, VII, and VIII, uniform rotation is introduced, and we have a discussion of the figures of equilibrium of rotating homogeneous liquids: Maclaurin's ellipsoid, Jacobi's ellipsoid with three unequal axes, Poincaré's pear-shaped figure, etc. The foot-notes give welcome references to the more recent literature. All this is interesting, but how much connection with geodesy it has is not so clear; the author does not discuss the possible cosmogonic implications, which are usually considered to constitute the most interesting applications of these theoretical developments.

Chapter IX treats Clairaut's problem, the figure of equilibrium and the form of the level surfaces for a slowly rotating mass of self-attracting liquid. The outer surface and the level surfaces within are approximately ellipsoids of revolution. Clairaut's differential equation connecting the ellipticity with the equatorial radius of any given level surface is established and integrated for several well known laws of density that make the results mathematically tractable. These laws suppose continuous variation in density from center to free surface. Seismologists look with more favor on discontinuities in density, but this physically interesting case is passed over, except for references to the literature. Radau's ingenious transformation of Clairaut's equation is given;