ON THE EXPANSION COEFFICIENTS OF THE FUNCTIONS $u/\mathrm{sn}u$ AND $u^2/\mathrm{sn}^2 u$ †

BY R. D. JAMES‡

1. *Introduction.* It is well known that the elliptic function $\mathrm{sn}(u, k)$ may be expanded in the form

$$\mathrm{sn}(u, k) = \sum_{n=0}^{\infty} \frac{u^n}{n!} S_n(k^2),$$

where

$$(1)\quad S_0(k^2) = 0, \quad S_1(k^2) = 1, \quad S_n(k^2) = \sum_{r=0}^{[(n-1)/2]} s_r(n) k^{2r}, \quad (n \geqq 2).$$

Moreover, $s_r(2j) = 0$ for all values of r and j, and $s_r(2j+1)$ is an integer >0 for $0 \leqq r \leqq j$. Similarly we have

$$(2)\qquad \frac{u}{\mathrm{sn}(u, k)} = \sum_{n=0}^{\infty} \frac{u^n}{n!} G_n(k^2),$$

where

$$G_0(k^2) = 1, \qquad G_n(k^2) = \sum_{r=0}^{[n/2]} g_r(n) k^{2r}, \qquad (n \geqq 1),$$

with $g_r(2j+1) = 0$ for all values of r and j, and $g_r(2j) \neq 0$ for $0 \leqq r \leqq j$. In particular

$$(3)\qquad g_0(2) = g_1(2) = 1/3.$$

Again,

$$(4)\qquad \frac{u^2}{\mathrm{sn}^2(u, k)} = \frac{1}{2} k^2 u^2 + \sum_{n=0}^{\infty} \frac{u^n}{n!} T_n(k^2),$$

where

$$T_0(k^2) = 1, \qquad T_n(k^2) = \sum_{r=0}^{[n/2]} t_r(n) k^{2r}, \qquad (n \geqq 1),$$

with $t_r(2j+1) = 0$ for all r and j, and $t_r(2j) \neq 0$ for $0 \leqq r \leqq j$. In particular,

$$(5)\quad t_0(2) = 2/3, \quad t_1(2) = -1/3, \quad t_0(4) = -t_1(4) = t_2(4) = 8/5.$$

---

† Presented to the Society, March 30, 1934.

‡ National Research Fellow.