systems, each of which violates the like-numbered postulate and satisfies all the other postulates of the set. These examples are all *arithmetic* systems, the elements being the numbers 0 and 1. The symbol $f(a, b)$ (mod 2) in an arithmetic system denotes the *least positive residue modulo* 2 obtained from $f(a, b)$ by rejecting multiples of 2. The operations $+$ and $\times$ are to be interpreted as the operations of ordinary arithmetic when they occur in the modular expression, otherwise they are to be interpreted as logical addition and logical multiplication.

EXAMPLE	K	$a+b$	ab
1	0, 1	$a+b+0/ab+1$ (mod 2)	ab (mod 2)
2	0, 1	ab (mod 2)	$a+b+0/ab+1$ (mod 2)
3	0, 1	$a+b$ (mod 2)	ab (mod 2)
4	0, 1	ab (mod 2)	$a+b$ (mod 2)
5	0, 1	ab (mod 2)	ab (mod 2)
6	Null		

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ON ANALYTIC FUNCTIONS WITH POSITIVE IMAGINARY PARTS

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The purpose of this paper is to give an integral representation of a function analytic in a half-plane, and with positive imaginary part there. This can be used to obtain in a simple way the well known analytic representation of the resolvent of a self-adjoint transformation in abstract Hilbert space.

THEOREM. *Let $\phi(l)$ be a function analytic for $\Im(l)>0$.*† *If*

$$(1) \qquad \Im[\phi(l)] \geqq 0, \qquad \limsup_{t\to\infty} |t\Im[\phi(it)]| < \infty,$$

for t real, $t>0$, then there is a uniquely determined monotone non-decreasing function $\alpha(\lambda)$, defined for $-\infty<\lambda<\infty$, satisfying

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† If ξ is any complex number, $\Re(\xi)$ and $\Im(\xi)$ will be used to denote its real and imaginary parts, and $\bar{\xi}$ its conjugate.