SHORTER NOTICES

Lehrbuch der höheren Mathematik für Universitäten und Technische Hochschulen. Volume III. Bearbeitet nach den Vorlesungen von Dr. Gerhard Kowalewski. Berlin and Leipzig, de Gruyter, 1933. 252 pp.

Much of this volume is devoted to a continuation of the study of differential and integral calculus begun in volume 2. A careful and precise development is made of some of the more advanced topics of calculus. For example, in the discussion of higher derivatives and differentials the author develops in turn Newton's first, second, . . . , nth difference quotients, thus obtaining the coefficients of the polynomial of degree n which coincides with $f(x)$ for $(n+1)$ distinct values of the independent variable. From this approximation polynomial he obtains John Bernoulli's integral form of the error term in Taylor's formula and conversely. By means of Taylor's formula analytical conditions for a maximum or minimum of a function of a single variable are obtained. There follows an excellent discussion of Taylor's formula with the customary developments of transcendental functions, algebraic functions, Newton's binomial series, functions defined by integrals, and power series.

The chapter dealing with integration of certain classes of functions is well written. A complete exposition of the integration of rational functions of a single variable leads to the treatment of elliptic integrals with an application to the pendulum problem and a brief note on hyperelliptic integrals.

In the section on differentials of functions of several variables, the author recurs to the vector space notions introduced in volume 1, viz., the vector $r=xi+yj+zk$ (Ortsvektor) from the origin to the point (x, y, z) and the displacement vector $idx+jdy+kdz=dr$. This leads to Taylor's theorem for several variables and to the discussion of maxima and minima of functions of several variables, implicit functions, and maxima and minima with neighboring conditions. The first chapter closes with an indication of the geometric meaning of differentials, a treatment of multiple integrals, the general Gauss integral theorem, from which are derived among other things the theory of the Amsler planimeter, and finally, some geometric applications of double integrals.

This first chapter comprises 163 of the 252 pages of the book and in it the material is developed with elegance, with precision, and with sufficient detail to make it interesting and illuminating reading for one who has a fair understanding of the elementary notions of the calculus.

In the remaining four chapters on differential equations, differential geometry, functions of a complex variable, and some problems of the calculus of variations, the author has not attempted to build up the foundations of a theory, but has contributed his own development of a few of the interesting problems which arise in these fields. They are chapters to be read with profit by the expert and may be expected to supply to a lecturer a new perspective and new ideas as to presentation.

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