NOTE ON A SINGULAR INTEGRAL*

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This note is concerned with the convergence in the mean of order 2, as $m \to \infty$, of the integral

$$(2\pi)^{-1/2} \int_{-\infty}^{+\infty} f(u) K(x - u; m) du.$$

We shall consider necessary and sufficient conditions for the convergence of the integral first to $f(x)$ and secondly to zero. We shall restrict ourselves in this paper to the case where $f(u)$ is of class $L_2(-\infty, +\infty)$, and $K(u; m)$ is of class $L_2(-\infty, +\infty)$ for all values of m. We introduce the following notation:

(a) $f(x) \subset L_p(-\infty, +\infty)$ if and only if $f(x)$ is measurable, and

$$\int_{-\infty}^{+\infty} |f(x)|^p dx < +\infty.$$

(b) Put

$$\|f(x)\|_p \equiv \left[\int_{-\infty}^{+\infty} |f(x)|^p dx \right]^{1/p}.$$

Then the statement "$f_m(x)$ converges to $f(x)$ in the mean of order p as $m \to \infty$" can be written "$\|f_m(x) - f(x)\|_p \to 0$ as $m \to \infty$." We shall also write

$$f(x) = \underset{p}{\text{l.i.m.}}\, f_m(x).$$

(c) Denote by $T[f(x)]$ the Fourier transform of $f(x)$. That is, if $f(x) \subset L_p(-\infty, +\infty)$, $p > 1$, then as $A \to \infty$,

$$T[f(x)] = \underset{p'}{\text{l.i.m.}}\, (2\pi)^{-1/2} \int_{-A}^{A} e^{-ixs} f(s) ds,$$

and $T[f(x)] \subset L_{p'}(-\infty, +\infty)$, where $1/p + 1/p' = 1$. The inverse operator T^{-1} is given by the relation

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