WHITEHEAD AND RUSSELL'S THEORY OF DEDUCTION AS A NON-MATHEMATICAL SCIENCE

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1. *Introduction.* In his paper in the June, 1931, issue of this Bulletin, B. A. Bernstein attempted to transform the theory of logic in Whitehead and Russell's *Principia Mathematica* into a mathematical science.† In this paper, I wish to discuss the general question whether or not this theory can be stated as a mathematical science. I shall use Bernstein's exposition as a guide in the discussion, because it not only states in admirable form and simplicity the nature of a mathematical science but also either brings up explicitly or at least suggests each of the issues involved.

2. *Nature of a Mathematical Science.* In the first place I shall summarize his account of the nature of a mathematical science, by which he says he means a pure deductive theory.

(1) It is "a body of propositions consisting of postulates and theorems."

(2) These propositions "give information about a certain class of *elements* and about certain *operations* or *relations* among the elements."

(3) "The classes, operations, and relations constitute the *ideas* of the science," some of which are taken as primitive, and the others of which are defined in terms of the primitive ones.

(4) "Every proposition must contain, besides the ideas *belonging to* the science, also ideas that are *outside* the science." This is necessary in order that the propositions may give information about the ideas within the science. The ideas outside the science are those of "general language".

(5) "Since the theorems are derived from the postulates, the science must use, beside the propositions belonging to it, also

† *Whitehead and Russell's theory of deduction as a mathematical science.* In vol. 35, No. 1, pp. 301–303, of the Transactions of this Society, E. V. Huntington made a similar attempt to mathematicize the logic of the *Principia.* His result is subject to the same general criticisms as I present in this paper against Bernstein's transformation.