AN INVOLUTORIAL LINE TRANSFORMATION DETERMINED BY A BILINEAR CONGRUENCE OF TWISTED ELLIPTIC QUARTIC CURVES*

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1. *Introduction.* Let there be given two elliptic space quartic curves α, β, bases, respectively, of the two pencils of quadrics $H_1-\alpha H_2=0$, and $K_1-\beta K_2=0$. The curve $C_4(\alpha,\beta)$ of intersection of a quadric of one pencil with one of the other meets each of α,β in 8 points. As the parameters α, β take on all values independently, $C_4(\alpha,\beta)$ describes a system of ∞^2 (a congruence of) elliptic space quartics. Through an arbitrary point (u) passes just one $C_4(\alpha, \beta)$, namely that for which $\alpha=H_1(u)/H_2(u)$ and $\beta=K_1(u)/K_2(u)$.

A quadric of the system

$$(1) \qquad (H_1 - \alpha H_2) - \rho(K_1 - \beta K_2) = 0$$

is determined by three independent linear relations among α, β, ρ, hence by any three points of space. If these three points be chosen on a straight line t, then the quadric of (1) determined by the three points contains t as a generator. Thus t is a bisecant of every elliptic quartic lying on the quadric. But the values of α, β so determined fix a $C_4(\alpha, \beta)$ of the congruence and it lies on the quadric of (1). Hence an arbitrary line t of space is bisecant to just one $C_4(\alpha, \beta)$.

Now, let $\gamma\equiv\sum_{i=1}^{4}c_iz_i=0$ be an arbitrary fixed plane. Any line t meets γ in a point P. The quadric $Q(t)$ of (1) which contains t as a generator has another generator t' also passing through P, and t' is likewise bisecant to the $C_4(\alpha, \beta)$ determined by t. The line transformation $t\sim t'$ is involutorial and birational. It is the purpose of this paper to study this involution I.†

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* Presented to the Society, March 30, 1934.

† A brief synthetic outline, mostly without proofs, of parts of this paper is given by J. de Vries: *On an involution among the rays of space, which is determined by a bilinear congruence of twisted elliptical quartics*, Proceedings Koninklijke Akademie van Wetenschappen te Amsterdam, vol. 22 (1919), pp. 493–496.