

ON THE FINITENESS OF THE CLASS NUMBER IN A SEMI-SIMPLE ALGEBRA*

BY C. G. LATIMER

1. *Introduction.* Let A be a rational semi-simple algebra of order n and let G be a domain of integrity of order n in A , according to Dickson's definition. † In a recent paper, ‡ Miss Shover showed that if A is a division algebra, the number of classes of left ideals in G is finite. She used the definitions of an ideal and a class of ideals as given by MacDuffee. § We shall extend this result to any A . Since Miss Shover also showed that there is a one-to-one correspondence between the classes of left ideals and the classes of right ideals in G , it will be sufficient to prove the theorem for right ideals. By applying this result, we shall obtain a theorem on similar matrices.

Artin proved the finiteness of the right ideal class number for a maximal ordnung in A , using a different definition of an ideal and a class of ideals. || Every domain of integrity of order n is an ordnung. An ordnung is a domain of integrity of order n if and only if it contains the modulus of A . In particular, every maximal ordnung is a domain of integrity of order n . Every non-singular ideal according to MacDuffee is an ideal according to Artin and if β is an element and K is an ideal in G , MacDuffee's and Artin's definitions of their norms $N(\beta)$, $N(K)$ are the same.

2. *Proof of the Theorem.* The only place in her paper where Miss Shover employed the hypothesis that A is a division algebra was in obtaining, for left ideals, a result equivalent to the following, which was proved by Artin for the case where G is maximal.

LEMMA 1. *There is a positive number C , depending only on G , such that if K is a non-singular right ideal in G , there is an element β in K for which*

* Presented to the Society, December 27, 1933.

† *Algebren und ihre Zahlentheorie*, p. 155.

‡ This Bulletin, vol. 39 (1933), pp. 610-14.

§ Transactions of this Society, vol. 31 (1929), pp. 71-90.

|| *Abhandlungen, Mathematisches Seminar, Hamburg*, vol. 5 (1927), pp. 251-289.