AN INVOLUTORIAL LINE TRANSFORMATION*

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1. *Introduction.* Consider a non-singular quadric H, a plane π not tangent to H, and a point O on H but not on π. In the plane π take a Cremona involutorial transformation I_n of order n with fundamental points in general position (not necessarily on the curve of intersection of π and H). Project H from O upon π by the projection P. The point transformation PI_nP^{-1} is involutorial and leaves H invariant as a whole. A point A on $H\sim(P)$ B on π;† $B\sim(I_n)$ B'; $B'\sim(P^{-1})$ A' on H. Now an arbitrary line t, with Plücker coordinates y_i, $(i=1, \cdots, 6)$, meets H in two points A_1, A_2 which $\sim(PI_nP^{-1})$ A_1', A_2'. The line $A_1'A_2'\equiv t'$ shall be called the conjugate of t by the *line transformation* T, and we write $t\sim(T)t'$. Since the point transformation PI_nP^{-1} is involutorial, so will the line transformation T be involutorial.

2. *Order of the Transformation* T. The coordinates of the points A_1, A_2 in which t meets H are quadratic functions of y_i; the coordinates of B_1, B_2 are linear in the coordinates of A_1, A_2 and hence are also quadratic functions of y_i; the coordinates of B_1', B_2' are functions of degree n in the coordinates of B_1, B_2 and are therefore functions of degree $2n$ in y_i; finally A_1', A_2' have coordinates of degree $2n$ in y_i. The Plücker coordinates of a line are quadratic functions of the coordinates of two points which determine the line, and hence the Plücker coordinates x_i of t' are of degree $4n$ in y_i. Thus T is of order $4n$.

3. *The Singular Lines of* T. Denote by O_1, O_2 the points where the generators g_1, g_2 of H through O meet π. The points O_1, $O_2\sim(I_n)O_1'$, $O_2'\sim(P^{-1})Q_1$, Q_2. The line $t\equiv Q_1Q_2\sim(T)$ the entire plane field of lines (g_1g_2), since O_1, $O_2\sim(P^{-1})g_1$, g_2.

Any line t tangent to H meets H in two points coincident at A. The point $A\sim(PI_nP^{-1})$ A', and hence $t\sim(T)$ the pencil of tangents to H at A'.

Since $O\sim(P)$ the whole line $O_1O_2\sim(I_n)$ a curve ρ of order

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† The symbol $\sim(P)$ means "corresponds in the transformation P to."