ON THE SUMMABILITY OF DERIVED CONJUGATE SERIES OF THE FOURIER-LEBESGUE TYPE*

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1. *Introduction.* We assume throughout that the function $f(x)$ is integrable in the sense of Lebesgue and satisfies the periodicity condition $f(x+2\pi)=f(x)$; then the series

$$(1) \qquad \sum_{\nu=1}^{\infty} (-1)^{r/2}[\nu^r(a_\nu \sin \nu x - b_\nu \cos \nu x)], \qquad (r \text{ even}),$$

and

$$(2) \qquad \sum_{\nu=1}^{\infty} (-1)^{(r-1)/2}[\nu^r(a_\nu \cos \nu x + b_\nu \sin \nu x)], \qquad (r \text{ odd}),$$

where a_ν, b_ν are the Fourier coefficients, are defined to be the rth derived conjugate series of the Fourier-Lebesgue type.

In a paper published in 1931†, Bosanquet and Linfoot introduced a regular method of summation which is weaker than that of the Cesàro means of any order $\alpha>0$ and is defined as follows. The series $\sum a_\nu$ is said to be summable (α, β) to S, where either $\alpha>0$, or $\alpha=0$, $\beta\geqq 0$, if

$$\sum_{\nu<n}\left[B(1-\nu/n)^\alpha \log^{-\beta}\left(\frac{C}{1-\nu/n}\right)a_\nu\right] \rightarrow S, \quad \text{as} \quad n\rightarrow\infty,$$

for C sufficiently large,‡ where $B=(\log C)^\beta$.

The object of this paper is to apply the Bosanquet-Linfoot method of summation to the series (1) and (2).§

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† L. S. Bosanquet and E. H. Linfoot, *On the zero order summability of Fourier series*, Journal of the London Mathematical Society, vol. 6 (1931), pp. 117–126.

‡ They have shown that it is equivalent to say "for every $C>1$"; see L. S. Bosanquet and E. H. Linfoot, *Generalized means and the summability of Fourier series*, Quarterly Journal of Mathematics, Oxford series, vol. 2 (1931), pp. 207–229.

§ This method has been applied to Fourier series, the conjugate series and the rth derived Fourier series. See the two papers of Bosanquet and Linfoot given above and A. H. Smith, *On the summability of derived series of the Fourier-Lebesgue type*, Quarterly Journal of Mathematics, Oxford series, vol. 4 (1933), pp. 93–106.