the forms (10) become

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\Delta^{(i+j)}f(c)\,\eta_i\eta_j. \tag{11}$$

If $\eta_0=\eta_1=\cdots=\eta_{n-1}=0$, $\eta_n=1$, we see that

$$\Delta^{(2n)}f(c) \geqq 0.$$

Since $f(x)$ is continuous by hypothesis, we may apply Lemma 2 and deduce that $f(x)$ is analytic in $a<x<b$. In (11) replace η_i by η_i/δ and let δ approach zero. We thus obtain

$$\sum_{i=0}^{n}\sum_{j=0}^{n}f^{(i+j)}(c)\,\eta_i\eta_j \geqq 0,$$

and by Lemma 3, the function $f(x)$ has the form (9). This completes the proof of the theorem.

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ARITHMETIC AND IDEAL THEORY OF ABSTRACT MULTIPLICATION*

BY A. H. CLIFFORD

If we are given a ring R we may be called upon to answer the following two questions.

1. Is every element of R uniquely decomposable into prime elements?
2. If not can we introduce *ideal* elements into R such that the resulting system has this property?

Since these questions can be put in terms involving only the operation of multiplication, it is natural to attempt a solution in the same terms. We start, therefore, with a group-like system in which multiplication only is defined, namely a class S satisfying the following postulates:

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* A statement of definitions and results of a thesis done under Professors E. T. Bell and Morgan Ward at the California Institute of Technology.

(Added in proof.) I find that ovoid ideals were first discovered by I. Arnold, *Ideale in kommutativen Halbgruppen*, Recueil Mathématique, Moscou, vol. 36 (1929), pp. 401–407. Arnold proves Theorem 4 for regular ova (which he calls commutative semi-groups), with a slightly different normal ideal arithmetic.