AN EXISTENCE THEOREM FOR GENERALIZED PFAFFIAN SYSTEMS*

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1. *Introduction.* The present paper has for its object the extension of Cartan's existence theorem† for a pfaffian system to a system of equations whose left members are symbolic differential forms of arbitrary degrees. Although the proof of the general case involves no essentially new principle, the arrangement of the details given here appears simpler than in any proof previously published for the linear case‡ and at the same time rigorous. The manner of presentation makes clear the relationship of the general theorem (and, consequently, of Cartan's theorem) to the theory of systems of partial differential equations as developed by Riquier.

2. *The Existence Theorem.* The set of equations

$$\omega^\lambda = 0, \qquad (\lambda = 1, 2, \cdots, \rho), \tag{1}$$

in which ω^λ is a symbolic differential form of degree p_λ, $(0 < p_\lambda \leqq n)$, will be called a *generalized pfaffian system.*§ The ordinary pfaffian system is the special case in which every ω is linear.

For the sake of simplicity, consider only one form temporarily, and put

$$\omega = a_{i_1 i_2 \cdots i_p} dx^{i_1} dx^{i_2} \cdots dx^{i_p}, \tag{2}$$

where the a's are functions of the x's and are skew-symmetric

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† E. Cartan, Annales de l'École Normale, (3), vol. 18 (1901), pp. 241–311; E. Goursat, *Problème de Pfaff*, 1922, pp. 343–381.

‡ C. Burstin, Recueil Mathématique de la Société Mathématique de Moscou, vol. 37 (1930), pp. 13–21, has recognized that the proof given by Goursat for the linear case can be materially simplified. His proof, which is apparently open to objections, will be discussed in a subsequent note in connection with singular integral varieties.

§ If the proposed system contains a set S_0 of equations whose left members are of degree zero, elimination of some of the variables x by means of S_0 gives a system of the type considered.