ENUMERATIVE PROPERTIES OF r-SPACE CURVES*

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In the determination of enumerative properties of algebraic curves it is often convenient to decompose a given curve C^n of order n and genus p to be studied into a number of component curves the sum of whose orders is equal to n. We may decompose C^n in various ways but we find it most convenient to decompose it completely into n lines with $n-1+p$ incidences. We call the system formed by these n lines an n-line or a skew n-sided polygon Γ with $n-1+p$ vertices. To determine the enumerative properties of the given curve C^n, we, in this paper, determine certain enumerative properties of Γ and then interpret the results for C^n. We shall obtain in this manner a number of results for C^n some of which are already well known and the others are less well known or are new.

Let the symbol $\{n\}_{x_1 x_2 \cdots x_q}^{(s)}$ denote the number of groups each consisting of $x_1+x_2+\cdots+x_q$ sides which are arranged in q sets such that each set contains x_i consecutive sides and that any two sets are separated by at least s consecutive sides not contained in them. Thus, $\{n\}_{11}^{(1)}$ means the number of pairs of non-consecutive sides of Γ. If $q=1$, we have $\{n\}_{x_1}^{(s)}$ or just $\{n\}_{x_1}$ which is the number of groups each of x_1 consecutive sides. The symbol $\{n\}^{(s)}$ or $\{n\}$ means the number of groups each containing no members and is therefore equal to unity. Hence,

$$(1) \qquad \{n\}^{(s)} = \{n\} = 1.$$

The following formula can be easily verified or can be proved by the method used below:

$$(2) \qquad \{n\}_{x_1}^{(s)} = \{n\}_{x_1} = n - (x_1 - 1) + (x_1 - 1)p.$$

The number of groups each consisting of q pairs of intersecting sides (or the number of groups of q non-consecutive vertices) of Γ is known† and is given by

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† This result is given without proof by B. C. Wong, *On loci of* $(r-2)$*-spaces incident with curves in r-space*, this Bulletin, vol. 36 (1930), pp. 755–761.