LAGRANGE MULTIPLIERS FOR FUNCTIONS OF INFINITELY MANY VARIABLES*

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The purpose of this note is to extend the Lagrange multiplier theorem to the case of a maximum of a function of infinitely many variables subject to an infinity of auxiliary conditions. The underlying implicit function theorems used are due to Hart.† The proof employs two lemmas on normal determinants and associated linear systems of equations which seem to have been overlooked.‡ One of these incidentally renders one assumption in Hart's implicit function theorem redundant.

LEMMA 1. *If* $\sum_{i,k}|a_{ik}| = A$ *and* A_{ik} *is the minor of* $\delta_{ik}+a_{ik}$ *in the determinant* $\Delta = |\delta_{ik}+a_{ik}|$, *then* $\sum_{i,k}|A_{ik}|$ $(i\neq k)$ *converges and the* $|A_{ii}|$ *are bounded.*

PROOF. Since $\sum_{i,k}|a_{ik}|$ converges, $\prod_k(1+\sum_i|a_{ik}|)$ converges. If $p=a_{i_1i_2}a_{i_2i_3}\cdots a_{i_ni_1}$, the infinite product $\prod(1+|p|)$ extended over all values of p is dominated by the product $\prod_k(1+\sum_i|a_{ik}|)$ and converges to a value P. A term of A_{ik}, $(i\neq k)$, has one of the forms

$$a_{ki}T, \qquad a_{ki_1}a_{i_1i_2}\cdots a_{i_ni}T,$$

where T is a product of factors p and the indices are all distinct. Hence

$$|A_{ik}| \leqq P\Big\{ |a_{ki}| + \sum_n \sum_{i_1\cdots i_n} | k, i_1, i_2, \cdots, i_n; i| \Big\},$$

where $|k, i_1, i_2, \cdots, i_n; i|$ is $|a_{ki_1}, a_{i_1i_2}, \cdots, a_{i_ni}|$ or zero according as the indices are distinct or not. Now

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† W. L. Hart, *Differential equations and implicit functions in infinitely many variables*, Transactions of this Society, vol. 18 (1917), Theorems XII, XIII, VI.

‡ For the normal determinant theory, see F. Riesz, *Les Systèmes d'Equations Linéaires . . .*, 1913.