SOME DEFINITE INTEGRALS INVOLVING SELF-RECIPROCAL FUNCTIONS

BY BRIJ MOHAN MEHROTRA

1. *Introduction.* In one of his papers, Ramanujan* has proved formally that if

$$\phi_\omega(t) = \int_0^\infty \frac{\cos \pi t x}{\cosh \pi x} e^{-\pi\omega x^2} dx,$$

then

$$(1) \qquad \phi_\omega(t) = \frac{e^{-\pi t^2/(4\omega)}}{\omega^{1/2}} \phi_{1/\omega}\left(\frac{it}{\omega}\right).$$

An examination of the proof shows that it rests on the fact that $\operatorname{sech}[x(\pi/2)^{1/2}]$ is self-reciprocal for cosine-transforms. The present investigation was suggested by this fact. The object of this note is to obtain a generalization of (1).

Following Hardy and Titchmarsh, I will say that a function is R_ν if it is its own J_ν transform, and it is $-R_\nu$ if it is skew-reciprocal for J_ν transforms; also, for $R_{1/2}$ and $R_{-1/2}$, I will write R_s and R_c, respectively.

2. THEOREM 1. *If*

$$\phi_\omega(t) = \omega^{1/2} \int_0^\infty e^{-\omega^2 x^2/2} f(x) \cos t\omega x \, dx,$$

where $f(x)$ *is* R_c *and is such that* $\int_0^\infty |f(x)|\, dx$ *converges, then*

$$(2) \qquad \phi_\omega(t) = e^{-t^2/2} \phi_{1/\omega}(it).$$

We have

$$(3) \quad \phi_\omega(t) = \left(\frac{2\omega}{\pi}\right)^{1/2} \int_0^\infty e^{-\omega^2 x^2/2} \cos t\omega x \, dx \int_0^\infty f(y) \cos xy \, dy.$$

This double integral is absolutely convergent, as we see by comparison with

* Ramanujan, *Some definite integrals*, Collected Papers, Cambridge University Press, 1927, pp. 202–207.