NOTE ON THE MOMENTS OF A BERNOULLI DISTRIBUTION*

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If p and $q=1-p$ denote, respectively, the probabilities of the success and the failure of an event in a single trial, then

$$f(x) = \frac{n!}{x!\,(n-x)!} q^{n-x}p^{x}, \qquad (x = 0, 1, \cdots, n),$$

is the probability of exactly x successes in n trials provided p is held constant throughout the n trials. A frequency distribution whose relative frequencies are given in accord with this law of probability is sometimes called a Bernoulli distribution.

The moments (per unit frequency) of a frequency distribution have long been regarded as useful characteristics of the distribution. We shall denote the sth moment about the vertical axis through the origin by μ_s', while the corresponding moment about the arithmetic mean will be denoted by μ_s. Thus, for the distribution given above, $\mu_s' = \sum_{x=0}^{n} x^s f(x)$, and, since the arithmetic mean is μ_1', $\mu_s = \sum_{x=0}^{n} (x-\mu_1')^s f(x)$. While it is easy to see that $\mu_1' = np$, it is not so easy to see what the results are for s arbitrary. With respect to this problem Karl Pearson remarked in an editorial:† "A simple reduction formula for the moments of a binomial $(p+q)^n$ about its mean was sought in vain. After a good deal of energy had been spent, we believe that μ_s, being the sth moment about the mean,

$$\mu_s = \left[\frac{d^s}{dx^s}(qe^{px} + pe^{-qx})^n\right]_{x=0},$$

is, perhaps, the easiest expression for reaching these moment coefficients by successive differentiation." However, Romanovsky‡ has derived the recursion formula

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† Karl Pearson, Biometrika, vol. 12 (1918–1919), footnote, p. 270.

‡ V. Romanovsky, *Note on the moments of the binomial* $(p+q)^n$ *about its mean*, Biometrika, vol. 15 (1923), pp. 410–412. See also *Les Principes de la Statistique Mathématique*, 1933, pp. 39–40 and pp. 320–321.