VEBLEN ON PROJECTIVE RELATIVITY

Projektive Relativitätstheorie. By O. Veblen. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 2, Heft 1. Berlin, Springer, 1933. 6+73 pp.

This book by Professor Veblen is a result of a series of lectures given at the University of Göttingen during the summer of 1932. It deals with that new aspect of the theory of relativity which is often called projective relativity on account of its relation to projective geometry. It allows a unified theory of the gravitational and the electromagnetic field, and, though it is not referred to in Veblen's book, also offers a possibility of including modern wave mechanics. Apart from this it throws a new light on such an old theory as classical projective geometry. The material is mainly taken from papers by the author himself and by close collaborators. The exposition is elegant and clear.

The present theory is therefore a result of two series of investigations, one mathematical and one physical. The mathematical side is the theory of projective connections, the physical side consists in the many attempts, begun by H. Weyl in 1918, to define a space-time structure depending not only on gravitational but also on electromagnetic potentials. Projective relativity seems to offer a rather simple and attractive solution.

The theory of projective connections is a generalization of projective geometry in the same sense as Riemannian geometry is a generalization of euclidean geometry. It is a theory of manifolds for which ordinary projective relations exist in the immediate neighborhood of a generating point, these relations being connected by a law which makes the manifold a "curved" projective manifold.

This is done in the following way. In a four-dimensional manifold with coordinates x^i, $i=1, 2, 3, 4$, there belongs to every point a "tangential space" of the dx^i which can be considered as an affine space. In a Riemannian geometry of fundamental tensor g_{ij} we have, at every point, a "light cone" $g_{ij}dx^i dx^j=0$ in the tangential space. We now introduce a non-degenerate quadric of which this cone is the asymptotic cone, and it is possible to define in each tangential space a non-euclidean geometry with respect to this quadric. The projective differential geometry of this kind of relativity is the "curved" generalization of this geometry.

To master the properties homogeneous coordinates are introduced into the space of the dx^i through the relations

$$dx^i = X^i/(\phi_\alpha X^\alpha), \quad (i = 1, 2, 3, 4; \alpha = 0, 1, 2, 3, 4);$$

the ϕ_α are functions which allow us to write for the hyperplane at infinity the equation $\phi_\alpha X^\alpha=0$. Homogeneous coordinates do not change when they are multiplied by a factor e^{x^0} of the coordinates (x^1, x^2, x^3, x^4). With respect to the transformations

$$\bar{x}^0 = x^0 + \log \rho, \quad \bar{x}^i = \bar{x}^i(x), \ (\rho \text{ a function of } x^1, \cdots, x^4)$$