A CYCLIC INVOLUTION OF ORDER SEVEN

BY W. R. HUTCHERSON

1. *Introduction.* In an earlier paper,† the writer discussed a cubic surface in ordinary three way space containing an involution of order five, I_5. This paper concerns itself with a different cubic surface which contains a cyclic involution, I_7.

2. *Discussion of I_7 Belonging to F_3 in S_3.* Consider the surface

$$F_3(x_1,x_2,x_3,x_4) \equiv ax_2^2x_3 + bx_3^2x_1 + cx_1x_2x_4 = 0$$

in S_3, invariant under the cyclic collineation T of order seven

$$x_1' : x_2' : x_3' : x_4' = x_1 : \epsilon x_2 : \epsilon^2 x_3 : \epsilon^3 x_4, \qquad (\epsilon^7 = 1).$$

There are four invariant points, $P_1 \equiv (1, 0, 0, 0)$, $P_2 \equiv (0, 1, 0, 0)$, $P_3 \equiv (0, 0, 1, 0)$, and $P_4 \equiv (0, 0, 0, 1)$. Each lies on the surface F, and since these are the only possible invariant points, the surface F has only four points of coincidence. It will be noticed, however, that only P_2 and P_3 are simple points of F. Hence this paper will not be interested in the two double invariant points, P_1 and P_4.

Consider a curve C, not transformed into itself by T, and passing through P_2. Take the plane $x_3 + \lambda x_4 = 0$ of the pencil passing through P_2 and P_1, tangent to C. This plane is transformed into

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P28 and its equivalent P6 are regarded as part of the "formal" theory; but both may be omitted, if preferred, without prejudice to the other postulates.)

What is perhaps the most obvious example of a "formal Principia system with equality" is the system (K, C, $+$, $'$, $=$) obtained from Example 0.4 by changing the word "correct" to "truistic." The resulting example satisfies all the Postulates P1–P6, P8–P11, but fails on P7 (since there are verdicts a such that neither a nor a' is a "truistic" verdict).

Thus the distinction between an "*informal* Principia system with equality" and a "*formal* Principia system with equality" depends on the inclusion or rejection of Postulate P7.

It is important to observe, however, that another, equally good, example of a "formal Principia system with equality" is the system obtained from Example 0.5 by changing the word "incorrect" to "absurd." The mathematical postulates by themselves give no precedence to the "truistic-or" interpretation over the "absurd-and" interpretation.

† W. R. Hutcherson, *Maps of certain cyclic involutions on two-dimensional carriers*, this Bulletin, vol. 37 (1931), pp. 759–765.