This determinant will vanish only when the points on the curve C corresponding to the values a_1, a_2, and a_3 lie on a straight line. When the determinant vanishes, we have $f(y)=f(y')$, and hence the points P and P' coincide.

If C is a straight line, the determinant vanishes identically and all curves have the closure property. If C is not cut by any straight line in more than two points, then none of the curves have the closure property.

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NOTE ON HOMOGENEOUS FUNCTIONALS*

BY L. S. KENNISON

The classical formula of Euler for functions homogeneous in n variables is as follows.

Let $f(x_1, \cdots, x_n)$ be a differentiable function of the n variables, $x_1, \cdots, x_n$, such that

$$f(\lambda x_1, \cdots, \lambda x_n) = \lambda^p f(x_1, \cdots, x_n). \tag{1}$$

Then we have

$$x_1 \frac{\partial f}{\partial x_1} + \cdots + x_n \frac{\partial f}{\partial x_n} = pf(x_1, \cdots, x_n). \tag{2}$$

The following analog of this formula for functionals of one variable was proved by E. Freda.†

Let $F|[f(x)]|$ be a functional with a Fréchet differential $\delta F=\int_0^1 F' \mid [f(x)];\ \xi \mid \delta f(\xi)d\xi+\sum_1^n A_s \mid [f(x)] \mid \delta f(x_s)$, where $x_1, \cdots, x_n$ are points of the interval $(0, 1)$, and such that

$$F \mid [\lambda f(x)] \mid = \lambda^r F \mid [f(x)] \mid.$$

Then

$$\left\{\frac{\partial}{\partial\lambda} F \mid [f(x)(1+\lambda)] \mid \right\}_{\lambda=0} = rF \mid [f(x)] \mid.$$

Theorem 2 of this paper will be a generalization of this theorem of Freda.

The following theorem is classical.

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† Rendiconti dei Lincei, (5), vol. 24 (1915), p. 1035.