ON THE SUMMATION OF FOURIER SERIES

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1. *Introduction.* Suppose that we have a series

$$(1)\qquad u_1 + u_2 + u_3 + \cdots$$

and an infinite matrix $\{\eta_{n,m}\}$, $(0 \leqq n < \infty,\ 1 \leqq m < \infty)$. We say that the series is summable by the method $\{\eta\}$, if the sum

$$(2)\qquad \sum_{n=0}^{\infty} u_n \eta_{n,m}$$

tends to a limit as m tends to infinity.

The method $\{\eta\}$ is said to be regular if, whenever (1) is convergent, (2) tends to the same limit.

Hille and Tamarkin* call a method of summability F-effective, if it sums the Fourier series of an integrable function to the value of the function at every point x where the function has a definite value $f(x)$ and where

$$\phi(t) \equiv f(x+t) + f(x-t) - 2f(x) = o(1),$$

as $t \rightarrow 0$. They call a method L-effective if it sums the Fourier series of an integrable function to the value of the function at every point of the Lebesgue set of the function; that is, at every point x where

$$\int_0^h |\phi(t)|\, dt = o(h).$$

The object of this paper is to construct a method of summation which is regular and F-effective without being L-effective.†

2. *Outline of the Method.* We restrict ourselves to methods of

* E. Hille and J. D. Tamarkin, *On the summability of Fourier series*, Transactions of this Society, vol. 34 (1932), pp. 757–783.

† This problem was proposed to the late R. E. A. C. Paley by Professors Hille and Tamarkin. Before his death, Paley had worked on the problem, leaving an unfinished and incomplete manuscript. On the basis of his work Randels and Rosskopf have proceeded and have been able to solve the problem.